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CONSTRUCTION OF EFFICIENCY BALANCED DESIGNS

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ABSTRACT: This paper deals with the methods of construction of binary and non-binary efficiency balanced (EB) designs which are based on the incidence matrices of balanced incomplete block (BIB) designs.

KEY WORDS: BIB design, EB design, binary and non-binary designs, augmented design, kronecker product of matrices.

1. INTRODUCTION

The concept of an EB design was first introduced by Jones (1959). However, he called such a design as a total balanced design. Later Puri and Nigam (1975a) renamed this total balanced design of Jones (1959) as efficiency balanced (EB) design. Calinski (1971) has discussed this concept in detail and gave some examples of EB designs by trial and error. Puri and Nigam (1975a) proved that a design is EB if all the off-diagonal elements of the matrix $P = NK^{-1}N'$ are all proportional to the product of the two relevant replications. Puri and Nigam (1975b) gave a systematic procedure of constructing EB designs by merging of treatments in BIB and EB designs. The usual concepts of variance balance and efficiency balance are special cases of the general definition of balance. Several authors studied the concept and gave many interesting properties and methods of construction of EB designs. To quote we mention few Williams (1975), Kageyama (1980), Dey et al. (1981), Dey and Singh (1980), Ghosh and Karmarkar (1988).

In what follows, we denote by $\otimes$ the kronecker product of matrices, $\mathbf{1}_x$ the column $x \times 1$ column vector of ones, $\mathbf{1}'_x$ the $1 \times x$ row vector of ones $\mathbf{1}'_x \otimes N$ the x replications of N, $O_{x\times y}$ the null matrix of order $x \times y$, I_x the identity matrix of order x, $J_{x\times y}$ the matrix of ones of order $x \times y$, and by x_l (l = 1, 2, 3), q the positive integers.

2. METHODS OF CONSTRUCTION OF EB DESIGNS

In this section, we describe methods of construction of binary and non-binary EB designs making use of the incidence matrices of some known BIB designs.

Theorem 2.1: Let N_1 be the $v_1 \times b_1$ incidence matrix of a BIB design D_1 with parameters $v_1, b_1, r_1, k_1, \lambda_1$. Then

$$N = \begin{bmatrix} x_1 N_1 & \vdots & \begin{matrix} x_2 I_{v_1-1} \\ x_3 \mathbf{1}'_{v_1-1+q} \end{matrix} & O_{(v_1-1)\times q} \end{bmatrix}$$

is the incidence matrix of an EB design D with parameters $v = v_1$, $b = b_1 + v_1 - 1 + q$, $\boldsymbol{r}' = [(x_1 r_1 + x_2)\mathbf{1}'_{v_1-1}, \{x_1 r_1 + x_3(v_1 - 1 + q)\}]$, $\boldsymbol{k}' = \{x_1 k_1 \mathbf{1}'_{b_1}, (x_2 + x_3)\mathbf{1}'_{v_1-1}, x_3 \mathbf{1}'_q\}$ and

$$E = x_1\lambda_1\{x_1 v_1 r_1 + x_2(v_1 - 1) + x_3(v_1 - 1 + q)\}/k_1(x_1 r_1 + x_2)^2$$

if and only if constant q satisfy the equality

$$x_1 x_3 \lambda_1 (x_2 + x_3) q = x_1(x_2 + x_3)\lambda_1\{x_2 - x_3(v_1 - 1)\} + x_2 x_3 k_1(x_1 r_1 + x_2).$$

Proof: Evidently the off-diagonal elements of the $C = (c_{ij})$ matrix of D are:

$$c_{ij} = \frac{x_1\lambda_1}{k_1} \qquad ; i, j \le v_1 - 1 \;\&\; i \ne j \tag{2.1}$$

$$c_{ij} = \frac{x_1\lambda_1}{k_1} + \frac{x_2 x_3}{(x_2 + x_3)} \qquad ; i \le v_1 - 1 \;\&\; j = v_1 \tag{2.2}$$

Now suppose D is an EB design. Then one has $c_{ij} = c r_i r_j$. Hence, considering the $c_{ij}'s$ as given above we have

$$\frac{x_1\lambda_1}{k_1} = c(x_1 r_1 + x_2)^2 \tag{2.3}$$

$$\frac{x_1\lambda_1}{k_1} + \frac{x_2 x_3}{(x_2 + x_3)} = c(x_1 r_1 + x_2)\{x_1 r_1 + x_3(v_1 - 1 + q)\} \tag{2.4}$$

Now from (2.3) and (2.4) eliminating c we get the required result. Using (2.3) we get efficiency E. Hence the proof.

Table 2.1: EB designs using Theorem 2.1

Sr. No.	Series No.	Parameters $v_1, b_1, r_1, k_1, \lambda_1$	x_1	x_2	x_3	q	Parameters of EB design $v, b, \boldsymbol{r}', \boldsymbol{k}'$	E
1	1	4,6,3,2,1	1	3	3	4	$4, 13, \{6\mathbf{1}'_3, 24\}, \{2\mathbf{1}'_6, 6\mathbf{1}'_3, 3\mathbf{1}'_4\}$	0.58
2	2	4,4,3,3,2	1	4	2	6	$4, 13, \{7\mathbf{1}'_3, 21\}, \{3\mathbf{1}'_4, 6\mathbf{1}'_3, 2\mathbf{1}'_6\}$	0.57

3	3	5,10,4,2,1	3	6	3	6	5,20, $\{18\mathbf{1}'_{4}, 42\}$, $\{6\mathbf{1}'_{10}, 9\mathbf{1}'_{4}, 3\mathbf{1}'_{6}\}$	0.53
4	4	5,5,4,4,3	2	6	2	6	5,15, $\{14\mathbf{1}'_{4}, 28\}$, $\{8\mathbf{1}'_{5}, 8\mathbf{1}'_{4}, 2\mathbf{1}'_{6}\}$	0.64
5	5	5,10,6,3,3	1	2	2	1	5,15, $\{8\mathbf{1}'_{4}, 16\}$, $\{3\mathbf{1}'_{10}, 4\mathbf{1}'_{4}, 2\mathbf{1}'_{1}\}$	0.75
6	6	6,15,5,2,1	2	8	4	9	6,29, $\{18\mathbf{1}'_{5}, 66\}$, $\{4\mathbf{1}'_{15}, 12\mathbf{1}'_{5}, 4\mathbf{1}'_{9}\}$	0.48
7	7	6,10,5,3,2	2	6	2	7	6,22, $\{16\mathbf{1}'_{5}, 34\}$, $\{6\mathbf{1}'_{10}, 8\mathbf{1}'_{5}, 2\mathbf{1}'_{7}\}$	0.59
8	8	6,6,5,5,4	2	2	1	2	6,13, $\{12\mathbf{1}'_{5}, 17\}$, $\{10\mathbf{1}'_{6}, 3\mathbf{1}'_{5}, 1\mathbf{1}'_{2}\}$	0.86
9	10	7,7,3,3,1	2	4	1	10	7,23, $\{10\mathbf{1}'_{6}, 22\}$, $\{6\mathbf{1}'_{7}, 5\mathbf{1}'_{6}, 1\mathbf{1}'_{10}\}$	0.55
10	11	7,7,4,4,2	1	2	2	1	7,14, $\{6\mathbf{1}'_{6}, 18\}$, $\{4\mathbf{1}'_{7}, 4\mathbf{1}'_{6}, 2\mathbf{1}'_{1}\}$	0.75
11	12	7,21,6,2,1	2	4	4	3	7,30, $\{16\mathbf{1}'_{6}, 48\}$, $\{4\mathbf{1}'_{21}, 8\mathbf{1}'_{6}, 4\mathbf{1}'_{3}\}$	0.56
12	13	7,7,6,6,5	2	8	4	4	7,17, $\{20\mathbf{1}'_{6}, 52\}$, $\{12\mathbf{1}'_{7}, 12\mathbf{1}'_{6}, 4\mathbf{1}'_{4}\}$	0.72
13	14	8,28,7,2,1	1	2	1	7	8,42, $\{9\mathbf{1}'_{7}, 21\}$, $\{2\mathbf{1}'_{28}, 3\mathbf{1}'_{7}, 1\mathbf{1}'_{7}\}$	0.52
14	15	8,14,7,4,3	6	6	2	4	8,25, $\{48\mathbf{1}'_{7}, 64\}$, $\{24\mathbf{1}'_{14}, 8\mathbf{1}'_{7}, 2\mathbf{1}'_{4}\}$	0.78
15	16	8,8,7,7,6	2	4	2	2	8,17, $\{18\mathbf{1}'_{7}, 32\}$, $\{14\mathbf{1}'_{8}, 6\mathbf{1}'_{7}, 2\mathbf{1}'_{2}\}$	0.84
16	17	9,12,4,3,1	3	4	4	1	9,21, $\{16\mathbf{1}'_{8}, 48\}$, $\{9\mathbf{1}'_{12}, 8\mathbf{1}'_{8}, 4\mathbf{1}'_{1}\}$	0.69
17	18	9,36,8,2,1	2	8	4	10	9,54, $\{24\mathbf{1}'_{8}, 88\}$, $\{4\mathbf{1}'_{36}, 12\mathbf{1}'_{8}, 4\mathbf{1}'_{10}\}$	0.49
18	19	9,18,8,4,3	2	2	1	2	9,28, $\{18\mathbf{1}'_{8}, 26\}$, $\{8\mathbf{1}'_{18}, 3\mathbf{1}'_{8}, 1\mathbf{1}'_{2}\}$	0.79

19	20	9,12,8,6,5	2	4	2	2	9,22, $\{20\mathbf{1}'_{8}, 36\}$, $\{12\mathbf{1}'_{12}, 6\mathbf{1}'_{8}, 2\mathbf{1}'_{2}\}$	0.82
20	21	9,9,8,8,7	2	12	4	7	9,24, $\{28\mathbf{1}'_{8}, 76\}$, $\{16\mathbf{1}'_{9}, 16\mathbf{1}'_{8}, 4\mathbf{1}'_{7}\}$	0.67
21	22	9,18,10,5,5	3	6	3	2	9,28, $\{36\mathbf{1}'_{8}, 60\}$, $\{15\mathbf{1}'_{18}, 9\mathbf{1}'_{8}, 3\mathbf{1}'_{2}\}$	0.81
22	23	10,15,6,4,2	1	4	4	2	10,26, $\{10\mathbf{1}'_{9}, 50\}$, $\{4\mathbf{1}'_{15}, 8\mathbf{1}'_{9}, 4\mathbf{1}'_{2}\}$	0.7
23	24	10,45,9,2,1	3	5	3	6	10,60, $\{32\mathbf{1}'_{9}, 72\}$, $\{6\mathbf{1}'_{45}, 8\mathbf{1}'_{9}, 3\mathbf{1}'_{6}\}$	0.53
24	25	10,30,9,3,2	2	4	2	4	10,43, $\{22\mathbf{1}'_{9}, 44\}$, $\{6\mathbf{1}'_{30}, 6\mathbf{1}'_{9}, 2\mathbf{1}'_{4}\}$	0.67
25	26	10,18,9,5,4	2	4	1	6	10,33, $\{22\mathbf{1}'_{9}, 33\}$, $\{10\mathbf{1}'_{18}, 5\mathbf{1}'_{9}, 1\mathbf{1}'_{6}\}$	0.76
26	27	10,15,9,6,5	1	6	6	1	10,25, $\{15\mathbf{1}'_{9}, 69\}$, $\{6\mathbf{1}'_{15}, 12\mathbf{1}'_{9}, 6\mathbf{1}'_{1}\}$	0.76
27	28	10,10,9,9,8	2	6	3	2	10,21, $\{24\mathbf{1}'_{9}, 51\}$, $\{18\mathbf{1}'_{10}, 9\mathbf{1}'_{9}, 3\mathbf{1}'_{2}\}$	0.82
28	29	11,11,5,5,2	4	8	2	8	11,29, $\{28\mathbf{1}'_{10}, 56\}$, $\{20\mathbf{1}'_{11}, 10\mathbf{1}'_{10}, 2\mathbf{1}'_{8}\}$	0.69
29	30	11,11,6,6,3	3	6	2	5	11,26, $\{24\mathbf{1}'_{10}, 48\}$, $\{18\mathbf{1}'_{11}, 8\mathbf{1}'_{10}, 2\mathbf{1}'_{5}\}$	0.75
30	31	11,55,10,2,1	2	6	6	4	11,69, $\{26\mathbf{1}'_{10}, 104\}$, $\{4\mathbf{1}'_{55}, 12\mathbf{1}'_{10}, 6\mathbf{1}'_{4}\}$	0.54
31	32	11,11,10,10,9	1	8	2	10	11,31, $\{18\mathbf{1}'_{10}, 50\}$, $\{10\mathbf{1}'_{11}, 10\mathbf{1}'_{10}, 2\mathbf{1}'_{10}\}$	0.64
32	33	11,55,15,3,3	2	6	3	4	11,69, $\{36\mathbf{1}'_{10}, 72\}$, $\{6\mathbf{1}'_{55}, 9\mathbf{1}'_{10}, 3\mathbf{1}'_{4}\}$	0.67

Theorem 2.2: Let N_L $(L = 1,2)$ be the $v_L \times b_L$ incidence matrix of a BIB design D_L with parameters v_L, b_L, r_L, k_L, λ_L. Then

$$N = \begin{bmatrix} x_1 N_1 & \vdots & \begin{matrix} x_2 N_{2(v_1-1)\times b_2} \\ x_3 \mathbf{1}'_{b_2+q} \end{matrix} & O_{(v_1-1)\times q} \end{bmatrix}$$

is the incidence matrix of an EB design D with parameters $v = v_1$, $b = b_1 + b_2 + q$, $\boldsymbol{r}' = [(x_1 r_1 + x_2 r_2)\mathbf{1}'_{v_{1-1}}, \{x_1 r_1 + x_3(b_2 + q)\}]$, $\boldsymbol{k}' = \{x_1 k_1 \mathbf{1}'_{b_1}, (x_2 k_2 + x_3)\mathbf{1}'_{b_2}, x_3 \mathbf{1}'_q\}$ and

$E = \frac{\{x_1\lambda_1(x_2k_2+x_3)+x_2{}^2\lambda_2 k_1\}\{x_1v_1r_1+x_2r_2(v_1-1)+x_3(b_2+q)\}}{k_1(x_2k_2+x_3)(x_1r_1+x_2r_2)^2}$ if and only if constant q satisfy the equality

$$x_3\{x_1\lambda_1(x_2k_2 + x_3) + x_2{}^2\lambda_2 k_1\}q$$

$$= x_1\lambda_1(x_2k_2 + x_3)(x_2r_2 - x_3b_2) + x_2k_1\{x_3r_2(x_1r_1 + x_2r_2) - x_2\lambda_2(x_1r_1 + x_3b_2)\}.$$

Proof: Evidently the off-diagonal elements of the $C = (c_{ij})$ matrix of D are:

$$c_{ij} = \frac{x_1\lambda_1}{k_1} + \frac{x_2{}^2\lambda_2}{(x_2k_2 + x_3)} \qquad ; i, j \le v_1 - 1 \;\&\; i \ne j \qquad (2.5)$$

$$c_{ij} = \frac{x_1\lambda_1}{k_1} + \frac{x_2x_3r_2}{(x_2k_2 + x_3)} \qquad ; i \le v_1 - 1 \;\&\; j = v_1 \qquad (2.6)$$

Now suppose D is an EB design. Then one has $c_{ij} = cr_ir_j$. Hence, considering the $c_{ij}{}'s$ as given above we have

$$\frac{x_1\lambda_1}{k_1} + \frac{x_2{}^2\lambda_2}{(x_2k_2 + x_3)} = c(x_1r_1 + x_2)^2 \qquad (2.7)$$

$$\frac{x_1\lambda_1}{k_1} + \frac{x_2x_3r_2}{(x_2k_2 + x_3)} = c(x_1r_1 + x_2)\{x_1r_1 + x_3(v_1 - 1 + q)\} \qquad (2.8)$$

Now from (2.7) and (2.8) eliminating c we get the required result. Using (2.7) we get efficiency E. Hence the proof.

Remark 2.1: In Theorem 2.2 if $x_3 = x_2\lambda_2/r_2$, then the design is also a VB design.

Table 2.2: EB designs using Theorem 2.2

Sr. No.	Series No. of N_1 and N_2	N_1 $(v_1,b_1,r_1,k_1,\lambda_1)$ N_2 $(v_2,b_2,r_2,k_2,\lambda_2)$	x_1	x_2	x_3	q	Parameters of EB design v, b, $\boldsymbol{r}'$, $\boldsymbol{k}'$	E
1	2	$N_1(4,4,3,3,2)$ $N_2(3,3,2,2,1)$	1	8	4	1	4,8, $\{19\mathbf{1}'_3, 19\}$, $\{3\mathbf{1}'_4, 20\mathbf{1}'_3, 4\mathbf{1}'_1\}$	0.81

2	3,1	$N_1(5,10,4,2,1)$ $N_2(4,6,3,2,1)$	1	3	1	3	$5,19, \{13\mathbf{1}'_4, 13\}, \{2\mathbf{1}'_{10}, 7\mathbf{1}'_6, 1\mathbf{1}'_3\}$	0.69
3	4,1	$N_1(5,5,4,4,3)$ $N_2(4,6,3,2,1)$	4	3	1	3	$5,14, \{25\mathbf{1}'_4, 25\}, \{16\mathbf{1}'_5, 7\mathbf{1}'_6, 1\mathbf{1}'_3\}$	0.86
4	6,3	$N_1(6,15,5,2,1)$ $N_2(5,10,4,2,1)$	3	4	1	6	$6,31, \{31\mathbf{1}'_5, 31\}, \{6\mathbf{1}'_{15}, 9\mathbf{1}'_{10}, 1\mathbf{1}'_6\}$	0.63
5	7,3	$N_1(6,10,5,3,2)$ $N_2(5,10,4,2,1)$	2	4	1	6	$6,26, \{26\mathbf{1}'_5, 26\}, \{6\mathbf{1}'_{10}, 9\mathbf{1}'_{10}, 1\mathbf{1}'_6\}$	0.72
6	7,5	$N_1(6,10,5,3,2)$ $N_2(5,10,6,3,3)$	1	4	2	2	$6,22, \{29\mathbf{1}'_5, 29\}, \{3\mathbf{1}'_{10}, 14\mathbf{1}'_{10}, 2\mathbf{1}'_2\}$	0.85
7	8,5	$N_1(6,6,5,5,4)$ $N_2(5,10,6,3,3)$	2	4	2	2	$6,18, \{34\mathbf{1}'_5, 34\}, \{10\mathbf{1}'_6, 14\mathbf{1}'_{10}, 2\mathbf{1}'_2\}$	0.89
8	10,6	$N_1(7,7,3,3,1)$ $N_2(6,15,5,2,1)$	2	5	1	10	$7,32, \{31\mathbf{1}'_6, 31\}, \{6\mathbf{1}'_7, 11\mathbf{1}'_{15}, 1\mathbf{1}'_{10}\}$	0.66
9	16,10	$N_1(8,8,7,7,6)$ $N_2(7,7,3,3,1)$	6	6	2	2	$8,17, \{60\mathbf{1}'_7, 60\}, \{42\mathbf{1}'_8, 20\mathbf{1}'_7, 2\mathbf{1}'_2\}$	0.93
10	14,11	$N_1(8,28,7,2,1)$ $N_2(7,7,4,4,2)$	2	4	2	1	$8,36, \{30\mathbf{1}'_7, 30\}, \{4\mathbf{1}'_{28}, 18\mathbf{1}'_7, 2\mathbf{1}'_1\}$	0.74
11	15,11	$N_1(8,14,7,4,3)$ $N_2(7,7,4,4,2)$	3	4	2	1	$8,22, \{37\mathbf{1}'_7, 37\}, \{12\mathbf{1}'_{14}, 18\mathbf{1}'_7, 2\mathbf{1}'_1\}$	0.87
12	16,11	$N_1(8,8,7,7,6)$ $N_2(7,7,4,4,2)$	2	8	4	1	$8,16, \{46\mathbf{1}'_7, 46\}, \{14\mathbf{1}'_8, 36\mathbf{1}'_7, 4\mathbf{1}'_1\}$	0.92
13	16,12	$N_1(8,8,7,7,6)$ $N_2(7,21,6,2,1)$	6	6	1	15	$8,44, \{78\mathbf{1}'_7, 78\}, \{42\mathbf{1}'_8, 13\mathbf{1}'_{21}, 1\mathbf{1}'_{15}\}$	0.81
14	19,14	$N_1(9,18,8,4,3)$ $N_2(8,28,7,2,1)$	2	7	1	21	$9,67, \{65\mathbf{1}'_8, 65\}, \{8\mathbf{1}'_{18}, 15\mathbf{1}'_{28}, 1\mathbf{1}'_{21}\}$	0.66
15	24,17	$N_1(10,45,9,2,1)$ $N_2(9,12,4,3,1)$	1	8	2	4	$10,61, \{41\mathbf{1}'_9, 41\}, \{2\mathbf{1}'_{45}, 26\mathbf{1}'_{12}, 2\mathbf{1}'_4\}$	0.72
16	23,22	$N_1(10,15,6,4,2)$ $N_2(9,18,10,5,5)$	1	2	1	2	$10,35, \{26\mathbf{1}'_9, 26\}, \{4\mathbf{1}'_{15}, 11\mathbf{1}'_{18}, 1\mathbf{1}'_2\}$	0.89
17	24,22	$N_1(10,45,9,2,1)$ $N_2(9,18,10,5,5)$	2	4	2	2	$10,65, \{58\mathbf{1}'_9, 58\}, \{4\mathbf{1}'_{45}, 22\mathbf{1}'_{18}, 2\mathbf{1}'_2\}$	0.8

18	25,22	$N_1(10,30,9,3,2)$ $N_2(9,18,10,5,5)$	4	2	1	2	$10,50, \{56\mathbf{1}'_9, 56\}$, $\{12\mathbf{1}'_{30}, 11\mathbf{1}'_{18}, 1\mathbf{1}'_2\}$	0.8
19	26,22	$N_1(10,18,9,5,4)$ $N_2(9,18,10,5,5)$	2	4	2	2	$10,38, \{58\mathbf{1}'_9, 58\}$, $\{10\mathbf{1}'_{18}, 22\mathbf{1}'_{18}, 2\mathbf{1}'_2\}$	0.9
20	28,22	$N_1(10,10,9,9,8)$ $N_2(9,18,10,5,5)$	3	2	1	2	$10,30, \{47\mathbf{1}'_9, 47\}$, $\{27\mathbf{1}'_{10}, 11\mathbf{1}'_{18}, 1\mathbf{1}'_2\}$	0.95
21	29,23	$N_1(11,11,5,5,2)$ $N_2(10,15,6,4,2)$	2	3	1	3	$11,29, \{28\mathbf{1}'_{10}, 28\}$, $\{10\mathbf{1}'_{11}, 13\mathbf{1}'_{15}, 1\mathbf{1}'_3\}$	0.86
22	30,23	$N_1(11,11,6,6,3)$ $N_2(10,15,6,4,2)$	1	3	1	3	$11,29, \{24\mathbf{1}'_{10}, 24\}$, $\{6\mathbf{1}'_{11}, 13\mathbf{1}'_{15}, 1\mathbf{1}'_3\}$	0.86
23	31,23	$N_1(11,55,10,2,1)$ $N_2(10,15,6,4,2)$	2	6	2	3	$11,73, \{56\mathbf{1}'_{10}, 56\}$, $\{4\mathbf{1}'_{55}, 26\mathbf{1}'_{15}, 2\mathbf{1}'_3\}$	0.74
24	33,23	$N_1(11,55,15,3,3)$ $N_2(10,15,6,4,2)$	2	9	3	3	$11,73, \{84\mathbf{1}'_{10}, 84\}$, $\{6\mathbf{1}'_{55}, 39\mathbf{1}'_{15}, 3\mathbf{1}'_3\}$	0.81
25	33,24	$N_1(11,55,15,3,3)$ $N_2(10,45,9,2,1)$	7	9	1	36	$11,136, \{186\mathbf{1}'_{10}, 186\}$, $\{21\mathbf{1}'_{55}, 19\mathbf{1}'_{45}, 1\mathbf{1}'_{36}\}$	0.67

Remark 2.2: In Table 2.1 and Table 2.2 series numbers are according to Raghavrao (1971), page 91.

3. CONCLUSION

A number of efficiency balanced designs generated by the new methods of construction given here. The designs so constructed are found to have applications in pharmaceutical, agricultural and industrial experiments. The methods are flexible enough to incorporate number of incidence matrices of BIB designs.

REFERENCES

1. Calinski, T. (1971): On some desirable patterns in block designs (with discussion), *Biometrics*, **27**, 275-292.
2. Dey, A. and Singh, M. (1980): Some series of efficiency-balanced designs, *Austral. J. Statist.*, **22**, 364-367.
3. Dey, A., Singh, M. and Saha, G.M. (1981): Efficiency balanced block designs, *Comm. Statist. - Theor. Meth.*, **10**, 237-247.
4. Ghosh, D.K. and Karmakar, P.K. (1988): Some series of efficiency-balanced designs, *Austral. J. Statist.*, **30**, 47-51.
5. Jones, R.M. (1959): On a property of incomplete blocks, *J. Roy. Statist. Soc. Ser. B*, **21**, 172-179.

6. Kageyama, S. (1980): On properties of efficiency-balanced designs, *Commun. Statist. - Theor. Meth.*, **9**, 597-616.
7. Puri, P.D. and Nigam, A.K. (1975a): On patterns of efficiency balanced designs, *J. Roy. Statist. Soc. Ser. B*, **37**, 457-458.
8. Puri, P.D. and Nigam, A.K. (1975b): A note on efficiency balanced designs, *Sankhya B*, **37**, 457-460.
9. Raghavrao, D. (1971): Construction and combinatorial problems in design of experiments, *John Wiley and Sons Inc.*, New York.
10. Williams, E.R. (1975): Efficiency balanced designs, *Biometrika*, **62**, 686-689.