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DECOMPOSITION OF RECURRENT CURVATURE TENSOR FIELDS IN A KAEHLERIAN MANIFOLD OF FIRST ORDER

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ABSTRACT: In this paper, we have defined and studied the decomposition of recurrent curvature tensor field in a Kaehlerian manifold of first order by considering the decomposition of recurrent curvature tensor in terms of a non-zero vector and a tensor fields. Also, several other theorems have been derived.

KEY WORDS: Riemannian space, Kaehlerian manifold, H-projective recurrent, curvature tensors.

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1. INTRODUCTION.

A 2n- dimensional Kaehlerian manifold $\mathbf{K^n}$ is a Riemannian space, if it admits a structure tensor F^h_i satisfying (Yano, 1965):

(1.1) $F^h_i F^j_h = -\delta^j_i$,

(1.2) $F_{ij} = -F_{ji}$, ($F_{ij} = F^a_i g_{aj}$) and

(1.3) $F^h_{i,j} = 0$,

Where the comma (,) followed an index denotes covariant differentiation with respect to the metric tensor g_{ij} of the Riemannian space.

The Riemannian curvature tensor field R^h_{ijk}, is given by

(1.4) $R^h_{ijk} = \partial_i \{{}_j{}^h{}_k\} - \partial_j \{{}_i{}^h{}_k\} + \{{}_i{}^h{}_l\} \{{}_j{}^l{}_k\} - \{{}_j{}^h{}_l\} \{{}_i{}^l{}_k\}$,

Whereas the Ricci tensor and the scalar curvature are respectively given by $R_{ij} = R^a_{aij}$ and $R = R_{ij} g^{ij}$. The Ricci tensor satisfies the following identities:

(1.5) $F^a_i R_{aj} = -R_{ia} F^a_j$,

(1.6) $F^a_i R^j_a = R^a_i F^j_a$.

(1.7) $F^i_a R^a_b F^b_j = -R^i_j$.

(1.8) $F^i_a R^a_i = -R^j_a F^a_j$.

(1.9) $F^a_i R^i_a = 0$.

If we now define a curvature tensor S_{ij} by

(1.10) $S_{ij} = -F^a_i R_{aj}$. Then we have

(1.11) $S_{ij} = -S_{ji}$. And

(1.12) $F^a_i S_{aj} = -S_{ia} F^a_j$.

It is well known that these tensors satisfy the identity given by (Tachibana 1967)

(1.13) $R^{a}_{ijk,\,a} = \ R_{jk,\,i} - R_{ik,\,j}$

The Kaehlerian holomorphically projective recurrent curvature tensor P^{h}_{ijk}, are given by (Sinha 1973)

(1.14) $$P^{h}_{ijk} = R^{h}_{ijk} + \frac{1}{n+2}\left(R_{ik}\,\delta^{h}_{j} - R_{jk}\,\delta^{h}_{i} + S_{ik}\,F^{h}_{j} - S_{jk}\,F^{h}_{i} + 2S_{ij}\,F^{h}_{k}\right),$$

The Bianchi identities in $\mathbf{K^n}$ are given by

(1.15) $R^{h}_{ijk} + R^{h}_{jki} + R^{h}_{kij} = 0$ and

(1.16) $R^{h}_{ijk\,,a} + R^{h}_{ika\,,j} + R^{h}_{iaj\,,k} = 0.$

Definition (1.1): A Kaehlerian space is said to be recurrent, if we have (Singh 1971)

(1.17) $R^{h}_{ijk,a} - \lambda_a R^{h}_{ijk} = 0,$

for some non-zero recurrence vector λ_a, and is called semi-recurrent (or Ricci-recurrent), if it satisfies

(1.18) $R_{ij,a} - \lambda_a R_{ij} = 0.$

Multiplying the above equation by g^{ij}, we get

(1.19) $R_{,a} - \lambda_a R = 0$

Remark (1.1): From (1.2) it follows that every Kaehlerian recurrent space is Kaehlerian Ricci-recurrent space but the converse is not necessarily true.

2. DECOMPOSITION OF RECURRENT CURVATURE TENSOR FIELDS IN A KAEHLERIAN MANIFOLD OF FIRST ORDER.

We consider the decomposition of recurrent curvature tensor field R^{h}_{ijk} in the following form (Singh, 1982):

(2.1) $R^{h}_{ijk} = X^{h}_{l}\, v^{l}\, \Phi_{ijk}\,,$

Where v^{l} is a non-zero vector field and X^{h}_{l}, Φ_{ijk} are two non-zero tensor fields such that (Negi and Gairola, 2010):

(2.2) $X^{h}_{l}\, \lambda_h = P_l$ and

(2.3) $\lambda_h\, v^{h} = 1$

P_l is called decomposed vector field and this is a non-zero vector field.

Definition (2.1): The vector field λ_a and the tensor field X^{h}_{l} given by equations (1.17) and (2.1) behave like recurrent vector and recurrent tensor fields and their recurrent forms are given by (Singh 1982)

(2.4) $\lambda_{a,m} = \mu_m\, \lambda_a$

and

(2.5) $X^{h}_{l,m} = v_m\, X^{h}_{l}\,,$

Where μ_m and v_m are non-zero recurrence vector fields.

Definition (2.2): Under the decomposition (2.1), the decomposed vector fields P_l is have like a recurrent vector field and its recurrent form is given by (Singh 1982)

(2.6) $$P_{l,m} = (\mu_m + v_m) P_l$$

We shall prove the following:

Theorem (2.1): Under the decomposition (2.1), the Bianchi identities for R^h_{ijk} take the forms (Takano, 1967)

(2.7) $$\Phi_{ijk} + \Phi_{jki} + \Phi_{kij} = 0, (\Phi_{ijk} = - \Phi_{ikj}) \quad \text{and}$$

(2.8) $$\lambda_a \Phi_{ijk} + \lambda_j \Phi_{ika} + \lambda_k \Phi_{iaj} = 0.$$

Proof: From equations (1.15), (1.16), (1.17) and (2.1), we get

(2.9) $$X^h_l v^l (\Phi_{ijk} + \Phi_{jki} + \Phi_{kij}) = 0 \quad \text{and}$$

(2.10) $$X^h_l v^l (\lambda_a \Phi_{ijk} + \lambda_j \Phi_{ika} + \lambda_k \Phi_{iaj}) = 0.$$

The identities (2.7) and (2.8) follow immediately from these equations and the fact $X^h_l v^l \neq 0$.

Theorem (2.2): Under the decomposition (2.1), the vector field v^l and the tensor field Φ_{ijk} be have like recurrent vector and recurrent tensor fields.

Proof: Multiplying equation (2.8) by $\mathbf{v^a}$ and using relation (2.3), we obtain

(2.11) $$\Phi_{ijk} = \lambda_k \Phi_{ij} - \lambda_j \Phi_{ik}$$

Where $\Phi_{ijk} v^k = \Phi_{ij}$ is a tensor fields.

Therefore, the relation (2.1) takes the form

(2.12) $$R^h_{ijk} = X^h_l v^l (\lambda_k \Phi_{ij} - \lambda_j \Phi_{ik})$$

Differentiating equation (2.12) covariantly with respect to x^m and using equations (1.17), (2.4), (2.5), (2.12), we get

(2.13) $$(\lambda_k \Phi_{ij} - \lambda_j \Phi_{ik}) v^l_{,m} = v^l [v_m(\lambda_j \Phi_{ik} - \lambda_k \Phi_{ij}) + \mu_m(\lambda_j \Phi_{ik} - \lambda_k \Phi_{ij})]$$

Multiplying this equation by v^a, we obtain

(2.14) $(\lambda_k \Phi_{ij} - \lambda_j \Phi_{ik}) v^a v^l_{,m} = v^l v^a [v_m(\lambda_j \Phi_{ik} - \lambda_k \Phi_{ij}) + \mu_m(\lambda_j \Phi_{ik} - \lambda_k \Phi_{ij})]$

This yield

(2.15) $v^a_{,} v^l_{,m} = v^l v^a_{,m}$

Since $v^l \neq 0$, there exists a proportional non-zero vector field π_m such that

(2.16) $v^l_{,m} = \pi_m v^l$

Therefore, v^l is recurrent vector field.

Further, differentiating equation (2.1) covariantly with respect to x^m and using equations (1.17), (2.1), (2.4), (2.5), (2.11), we obtain

(2.17) $\Phi_{ijk,m} = (\lambda_m - v_m - \pi_m)\Phi_{ijk}$

Hence, Φ_{ijk} is recurrent tensor field.

If $v_m + \pi_m \neq 0$, we have

Theorem (2.3): Under the decomposition (2.1), the vector field $X^h_l v^l$ is recurrent with the recurrence vector field $(v_m + \pi_m)$.

Proof: Differentiating the vector field $X^h_l v^l$ covariantly with respect to x^m and using equation (2.5) and (2.16), we get the proof.

On the other hand, if $v_m + \pi_m = 0$, we have

Theorem (2.4): Under the decomposition (2.1), Φ_{ijk} will be recurrent with the same recurrence vector λ_m as the curvature tensor field R^h_{ijk}.

Proof: The proof follows immediately from equation (2.17).

Theorem (2.5): Under the decomposition (2.1), the vector field v^l and tensor fields R^h_{ijk}, R_{ij}, Φ_{ijk} satisfying the relations

(2.18) $\lambda_h R^h_{ijk} = \lambda_i R_{jk} - \lambda_j R_{ik} = P_l v^l \Phi_{ijk}$

Proof: With the help of equations (1.13), (1.17) and (1.18), we obtain

(2.19) $\lambda_h R^h_{ijk} = \lambda_i R_{jk} - \lambda_j R_{ik}$

Multiplying equations (2.1) by λ_h and using relation (2.2), we obtain

(2.20) $\lambda_h R^h_{ijk} = P_l v^l \Phi_{ijk}$

From equations (2.19) and (2.20), we get the relations (2.18).

Theorem (2.6): Under the decomposition (2.1), the curvature tensor R^h_{ijk} and Kaehlerian holomorphically projective curvature tensor fields are equal iff

(2.21) $$\delta^h_j \Phi_{ik} - \delta^h_i \Phi_{jk} + \Phi_{ak} (F^h_j F^a_i - F^h_i F^a_j) + 2 F^h_k F^a_i \Phi_{aj} = 0.$$

Proof: Equation (1.14) may be expressed in the form

(2.22) $$P^h_{ijk} = R^h_{ijk} + D^h_{ijk}$$

where

(2.23) $$D^h_{ijk} = \frac{1}{n+2} (R_{ik} \delta^h_j - R_{jk} \delta^h_i + S_{ik} F^h_j - S_{jk} F^h_i + 2S_{ij} F^h_k),$$

Contracting indices **h** and **k** in (2.1), we obtain

(2.24) $$R_{ij} = X^k_l v^l \Phi_{ijk} ,$$

With the help of equation (2.24), we have

(2.25) $$S_{ij} = F^a_i R_{aj} = F^a_i X^r_l v^l \Phi_{ajr} ,$$

Making use of equations (2.24) and (2.25) in (2.23), we obtain

(2.26) $$D^h_{ijk} = \frac{X^r_l v^l}{(n+2)} [\Phi_{ikr} \delta^h_j - \Phi_{jkr} \delta^h_i + \Phi_{akr} (F^h_j F^a_i - F^h_i F^a_j) + 2 F^h_k F^a_i \Phi_{ajr}]$$

From equation (2.23), it is clear that $P^h_{ijk} = R^h_{ijk}$, iff $D^h_{ijk} = 0$, which in view of equation (2.26) becomes

(2.27) $$\Phi_{ikr} \delta^h_j - \Phi_{jkr} \delta^h_i + \Phi_{akr} (F^h_j F^a_i - F^h_i F^a_j) + 2 F^h_k F^a_i \Phi_{ajr} = 0.$$

Multiplying this equation by v^r and using the relation

$$\Phi_{ijk} v^k = \Phi_{ij} ,$$

We have the required equation.

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