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A production inventory model with random defective items under trade credit financing

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***Abstract**: This article illustrates an inventory model where production system is not perfect. As a consequence of this defective items are manufactured. Rework process is adopted on defective item to restore it into originals form. The ratio of imperfect item production rate and perfect item production rate is represented by a random variable. The random variable obeys uniform probability distribution. Trade credit policy also has been taken into consideration. A certain period is offered to the customer. Delay of payment for that period is permissible. Situations associated with interest paid and earn regarding credit period is considered. A mathematical model is formulated here. The aim of the model is to formulate cost function taking all cost components into consideration. Furthermore, it has been shown that the cost function may be minimized in order to optimize total cost of the system.*

***Keyword**: Inventory, imperfect production, rework, trade credit.*

Introduction:

Production is an essential part in an industry. But there are many issues on which production of a system depends. They may be raw material, functioning ability of machine, efficiency of labor etc. Violation of these things may turn a perfect production system into an imperfect one. Then defective items generate from faulty production system. It is the reality. For defective items there are two options. They may be reworked or may be treated as scrapped items. For costly material rework is better strategy from the end of manufacturer rather than making it into scrap. Another policy which is very common to the business world is trade credit policy. This policy is very much attractive as in this policy a time period is available by which payment can be delayed. Keeping all these things a production inventory model under the light of credit financing atmosphere is developed.

Literature Survey:

Imperfect production system attracts a lot to the researcher. There are many research works on this topic. First noticeable work on imperfect production system was done by Hayek and Salameh [1]. Importance of rework for defective items has been introduced by Cardenas-Barron [2] and [3]. In recent times extended work on imperfect production system was done by Al-Salameh [4]. The effect of inspection was incorporated in that model. Goyal [5] was pioneer to introduce trade credit policy in mathematical model. Abad and Jaggi [6] joint approach for selling unit price and the length of the credit period for a seller when demand is price sensitive. Pal et al. [7] considered two-stage credit system involving two members of a supply chain.

Notations:

P	Production rate
P_D	Production rate of defective items
γ	Demand rate
R	Reworking rate
T	Cycle length
Q	Lot Size
y	Fraction at which defective items are produced
M	Offered trade credit period
h	Holding cost per unit item for non-defective as well as defective items
C_{h_r}	Holding cost for reworked items
C_p	Production cost per unit item
C_r	Reworking cost per unit item
k	Set-up cost for each set up
$I(t)$	Inventory level for non-defective items at time t
$I_D(t)$	Inventory level for non-defective items at time t
I_E	Interest earn
I_P	Interest paid

Basic assumptions:

- A single stage model for single commodity is presented
- Defective items are produced at the rate P_D . $\frac{P_D}{P} = y$ where y is a continuous random variable follows uniform distribution with mean m and variance σ^2.
- During production time interval $\left[0, \frac{Q}{P}\right]$ demand of the customer is met by non-defective items only.
- After production process, reworking starts. All defective items are reworked.
- Demand rate γ is constant.
- A trade credit period M is offered. M Satisfies the following inequality. $M < T$.
- Defective item and non defective items have same holding cost.
- The decision variable is how much quantity to be produced.

Mathematical Model formulation:

We develop the model under two situations. Then those two situations are combined. First situation regarding inventory of non-defective items while second one is for defective items.

A. For non-defective items:

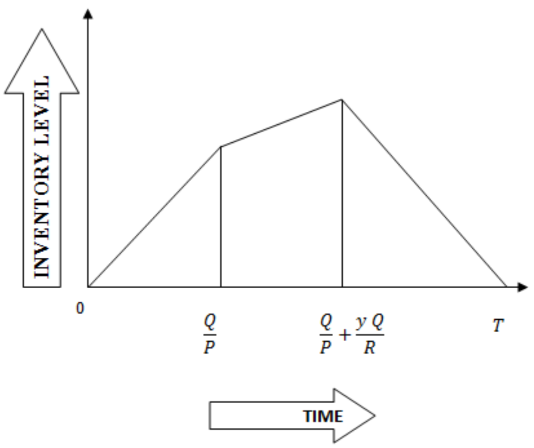

Fig 1: Time weighted inventory for non-defective items

Firstly, inventory of non-defective items increases at the rate $P - P_D - \gamma$ for the time interval $\left[0, \frac{Q}{P}\right]$. So the differential equation reflecting inventory level is given by

$$\frac{dI_1}{dt} = P - P_D - \gamma \text{ subject to } I\,(0) = 0 \qquad (1)$$

Solving this differential equation we get

$$I_1(t) = [(1-y)P - \gamma]\ t \qquad (2)$$

[Using the fact $\frac{P_D}{P} = y$]

Reworking process carried for the interval $\left[\frac{Q}{P}, \frac{Q}{P} + \frac{y\,Q}{R}\right]$. Since total amount produced is Q. So total defective amount produced is yQ. As the reworking rate is R, time needed for rework is $\frac{y\,Q}{R}$.

For the interval $\left[\frac{Q}{P}, \frac{Q}{P} + \frac{y\,Q}{R}\right]$ inventory grows up at the rate $R - \gamma$.

The differential equation for inventory level is

$$\frac{dI_2}{dt} = R - \gamma \text{ subject to } I_1\left(\frac{Q}{P}\right) = I_2\left(\frac{Q}{P}\right) \qquad (3)$$

Solving the differential equation is

$$I_2(t) = (R-\gamma)t + (1-y)Q - \frac{R\,Q}{P} \qquad (4)$$

In the interval $\left[\frac{Q}{P} + \frac{y\,Q}{R}, T\right]$ there is no rework as well as production. Inventory depletes at the rate $-\gamma$. The corresponding differential equation is

$$\frac{dI_3}{dt} = -\gamma \text{ with } I_3(T) = 0 \qquad (5)$$

Solving this, we have

$$I_3(T) = \gamma\,(T - t) \qquad (6)$$

Now cycle length is defined by

Cycle length = Amount consumed / demand rate. So the corresponding expression becomes

$$T = \frac{Q}{\gamma}\,. \qquad (7)$$

Cost for holding of non-defective items considering different time intervals

$$= h\left[\int_0^{\frac{Q}{P}} I_1(t)dt + \int_{\frac{Q}{P}}^{\frac{Q}{P}+\frac{y\,Q}{R}} I_2(t)dt + \int_{\frac{Q}{P}+\frac{y\,Q}{R}}^{T} I_3(t)dt\right]$$

$$= h\left[\frac{Q^2\{P\,R - (R + R\,y + P\,y^2)\gamma\}}{2\,P\,R\,\gamma}\right] \qquad (8)$$

[using (2), (4), (6) and (7)]

Here m and σ^2 represents mean and variance of the random variable y.

$E(y) = m \; and \; Var\,(y) = \sigma^2$

We know that $Var\,(y) = E(y^2) - \{E(y)\}^2$

So, $E(y^2) = m^2 + \sigma^2$.

Expected holding cost for non-defective items follows as

$$= h\left[\frac{Q^2\{P\,R - \left(R + R\ m + P\,(m^2+\sigma^2)\right)\gamma\}}{2\,P\,R\,\gamma}\right] \qquad (9)$$

B. For defective items

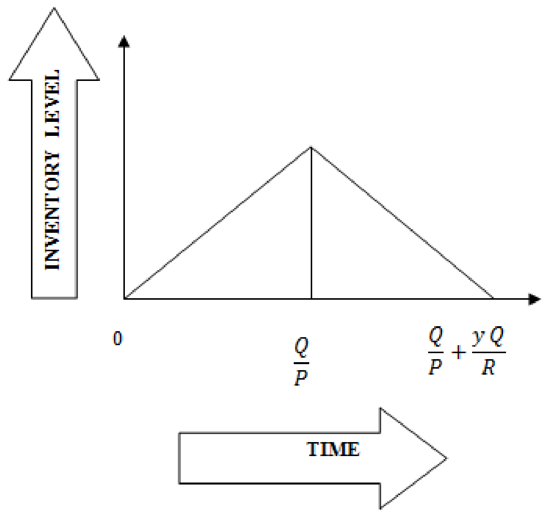

Fig 2: Time weighted inventory for defective items

Pilling of defective items occur at the rate P_D for the time interval $\left[0,\frac{Q}{P}\right]$ .Hence the governing differential equation is

$$\frac{dI_{D_1}(t)}{dt} = P_D \ \text{with}\ I_{D_1}(0) = 0 \qquad (10)$$

Solving this differential equation we get

$$I_{D_1}(t) = \gamma P t \qquad (11)$$

Then holding cost for defective items is given by

$$= h \int_0^{\frac{Q}{P}} I_{D_1}(t)\, dt = \frac{h\ Q^2\ \gamma}{2\ P}$$

Expected holding cost for defective items

$$= \frac{h\ Q^2\ E(\gamma)}{2\ P} = \frac{h\ Q^2\ m}{2\ P} \qquad (12)$$

Defective items reduces at the rate $-R$ for the time interval $\left[\frac{Q}{P}, \frac{Q}{P} + \frac{\gamma\, Q}{R}\right]$. Differential equation associated with this time interval is given by

$$\frac{dI_{D_2}(t)}{dt} = -R \ \text{subject to}\ I_{D_2}\left(\frac{Q}{P} + \frac{\gamma\, Q}{R}\right) = 0$$

This gives

$$I_{D_2}(t) = -R\, t + \frac{R\, Q}{P} + \gamma\, Q \qquad (13)$$

Holding cost for the item which is reworked is given by

$$C_{h_r} \int_{\frac{Q}{P}}^{\frac{Q}{P}+\frac{\gamma Q}{R}} I_{D_2}(t)\, dt = \frac{1}{2\ R} C_{h_r}\ Q^2\ \gamma^2$$

Expected holding cost for reworked items

$$= \frac{1}{2\ R} C_{h_r}\ Q^2\ \gamma^2 = \frac{1}{2\ R} C_{h_r}\ Q^2\ E(\gamma^2)$$

$$= \frac{1}{2\ R} C_{h_r}\ Q^2 (m^2 + \sigma^2) \qquad (14)$$

Another part is to be calculated for how much interest earned and paid.

C. Interest earn and paid

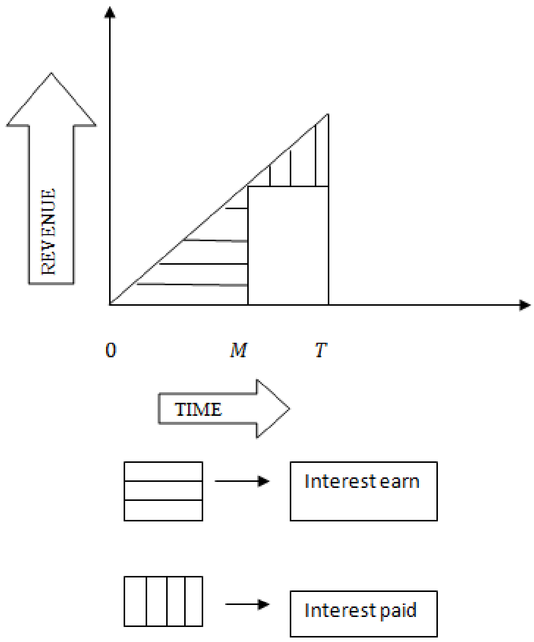

Fig 3: Time weighted revenue

Interest earn

$$= \frac{1}{2}\ \gamma\ I_E C_p\ M^2 \qquad (15)$$

Interest paid = $I_P C_p \int_M^T \gamma (t - M)\, dt$

$$= \frac{1}{2}\ I_P C_p\, \gamma\ (T - M)^2 \qquad (16)$$

Expected average total cost (ETC) of the system has several components: set up cost, production cost, holding cost for non-defective items, holding cost for defective items, holding cost for items that are reworked, cost of reworking, interest paid and interest earn.

$$ETC = \frac{1}{T}\Bigg[k + C_p\ Q + h\left\{\frac{Q^2\{P\ R - \left(R + R\ \ m + P\ (m^2+\sigma^2)\right)\gamma\}}{2\ P\ R\ \gamma}\right\} + \frac{h\ Q^2\ m}{2\ P} + \frac{1}{2\ R} C_{h_r}\ Q^2 (m^2 + \sigma^2)\ + C_r\ Q\ \gamma + \frac{1}{2}\ I_P C_p\, \gamma\ (T - M)^2 - \frac{1}{2}\ \gamma\ I_E C_p\ M^2\Bigg] \qquad (17)$$

Solution Procedure:

Our problem is to minimize ETC with respect to the lot-size (Q)

Differentiating (17) with respect to Q twice and using relation (7) we have

$$\frac{d^2}{dQ^2}\ (\text{ETC}) = \frac{1}{Q^3}\ [\gamma\{2\ k + C_p(I_P - I_E)\ M^2\ \gamma\}]$$

Normally interest paid (I_P) is larger than interest earn(I_E).
This implies
$\frac{d^2}{dQ^2}$ (ETC) > 0 as all other parameters are positive.
Then the average total cost of the system is minimized.

Conclusion:

In this work we have considered an imperfect production inventory model. Two scenarios related to non-defective items and defective items are portrayed and analyzed. Cost function has been developed considering various components of cost.
Also effect of trade credit in imperfect production system has been incorporated. We have been able to minimize cost function based on decision variable.
There are many options regarding extension of this model. This work can be extended for perishable items. Also shortage may be included.

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