

Determination of retailer's optimal pricing strategy under promotional effort dependent demand rate

SUBHANKAR ADHIKARI

DEPARTMENT OF MATHEMATICS, CHANDERNAGORE COLLEGE
CHANDERNAGORE, HOOGHLY, WEST BENGAL, INDIA, PIN-712136.

Abstract: This article illustrates an economic order quantity model. In this work demand rate is assumed to be dependent on promotional effort. and selling price. Another feature of this modeling is to include trade credit policy since it is an efficient tool for emerging business. We have calculated amount regarding interest earn and paid associated with permissible delay of payment. Two cases may appear as credit period lies or may exceed inventory cycle. A mathematical model has been presented here. The basic objective of this model is to determine maximum average profit along with determination of optimal price and promotional effort level. The proposed model is verified by a numerical example.

Keyword: Inventory, pricing, promotional effort, trade credit.

Introduction:

Due to globalization business world is getting more and more competitive day by day. Researchers are engaged in evaluating strategies on various issues associated with business. Pricing is one of the important issues among them. In fact what will be selling price of a commodity is a necessary question. In many cases price has to be forecast at the beginning of an inventory cycle. Moreover, promotion for selling a product helps to reach target of sales. There are many ways through which this can be done. Increase in the frequency of advertisement is one of them. Another option can be exercised by increasing number of efficient sales executive. Obviously there will be an extra cost regarding this. Now-a-day another tactics is frequently used in business strategy is delay in payment by offering trade credit. Consumers are attracted very much by this slogan "buy now and pay latter". In this work, a mathematical model has been formulated keeping all those aforesaid things.

Literature Survey:

Krishnan et al. [1] were first to incorporate the effect of promotional effort in inventory management and supply chain mechanism. Pal et al. [2] developed a two-echelon supply chain considering price, promotional effort and quality dependent demand rate. Pal et al. [3] incorporated criteria of promotional effort in shortage environment. Mandal and Pal [4] studied an

imperfect production inventory model with price and advertisement sensitive demand. How to demonstrate the effect of trade-credit in mathematical modeling was first shown by Goyal [5]. Tiwary et al. [6] studied impact of trade credit period as well as inflation for deteriorating items. Sarkar et al. [7] analyzed credit linked demand for perishable items whose deterioration rate depends on time. Two tier credit financing was done by Mandal et al. [8].

Notations:

D	Demand rate
s	Selling price
p	purchase price
c_s	Set up cost per unit
T	Cycle length
c_h	Holding cost
M	Offered trade credit period
ρ	Promotional effort
ξ	Exponent of promotional effort
$I(t)$	Inventory level for items at time t
I_o	Interest obtain
I_p	Interest paid
k	Cost coefficient associated with promotional effort
π_1	Average Profit for Case 1
π_2	Average Profit for Case 2

Basic assumptions:

- This frame work of modeling is based mainly to optimize profit from retailer’s point of view
- Supplier offer a time period M to the retailer for settlement of account
- After that period retailer has to pay interest to the supplier at the rate I_p . Retailer will enjoy interest from his revenue within the period M at the rate I_o
- Demand rate depends on both selling price and promotional effort. D being demand rate is a function of s and ρ . Explicit form of demand rate is taken as $D = D(\rho, s) = \rho^{\xi} + a - b s$ with selling price s satisfying the condition $p < s < \frac{a}{b}$ and promotional effort $\rho > 1$.
- This model has been framed for single product
- Time horizon is infinite
- Shortages are not allowed
- Cost of promotional effort is taken into account and its expression is $k \rho^2$

Model formulation:

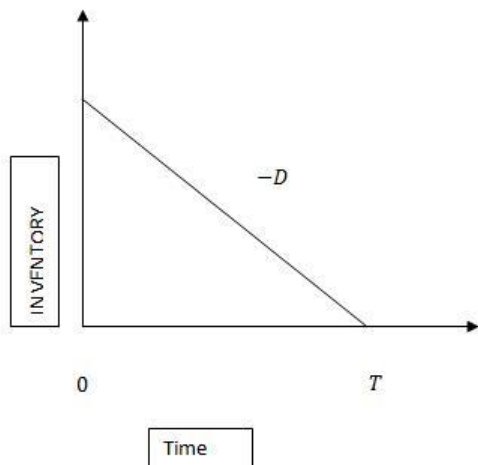


Fig1: Inventory level versus time

Inventory level depletes at the rate D . The differential equation governing inventory level is given by

$$\frac{dI}{dt} = -D \text{ subject to } I(T) = 0 \tag{1}$$

Solution of the differential equation is given below

$$I(t) = D(T - t) \tag{2}$$

Holding cost is expressed as

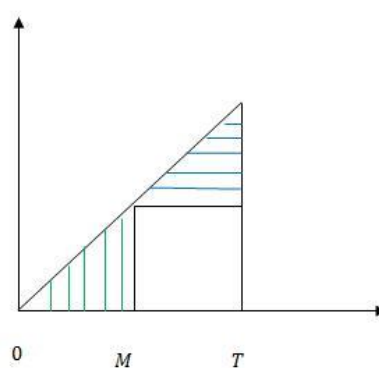
$$\begin{aligned} &= c_h \int_0^T I(t) dt \\ &= c_h \int_0^T D(T - t) dt \\ &= \frac{c_h}{2} D T^2 \end{aligned} \tag{3}$$

Now two possibilities may happen. Either trade credit period M lies within cycle length T or it may exceed T . Two cases thus comes provided below

Case 1: $T < M$

Case 2: $T \geq M$

We are now interested in calculating the amount regarding interest obtained and paid for every case.



Interest obtain area

Interest paid area

Fig 2: Accumulation of revenue with respect to time for case1

Now interest obtain for the period $[0, M]$ is

$$\begin{aligned} &= I_o s \int_0^M D t dt \\ &= \frac{1}{2} I_o s D M^2 \end{aligned} \tag{4}$$

It should be noted that interest obtain is determined on selling price while interest paid is determined on purchase price.

Interest has to be paid on left over inventory at time M

$$\begin{aligned} &= I_p p \int_M^T I(t) dt \\ &= \frac{1}{2} D (T - M)^2 I_p p \end{aligned} \tag{5}$$

For Case2

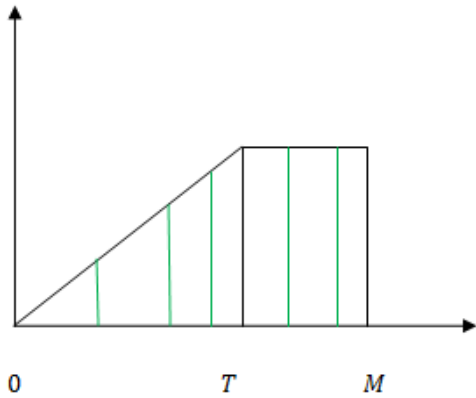


Fig3: Revenue collection with respect to time for Case2

In this Case as credit period exceeds cycle length .So no interest has to be paid. Retailer obtains interest only.

Interest obtain is given as follows

$$\begin{aligned}
 &= s I_o \left[\int_0^T Dt dt + \int_T^M DT dt \right] \\
 &= s I_o \left[\frac{1}{2} DT^2 + DT (M - T) \right] \\
 &= s I_o D T \left\{ \frac{T}{2} + M - T \right\} \\
 &= s I_o D T \left(M - \frac{T}{2} \right) \tag{6}
 \end{aligned}$$

Interest paid
 $= 0$ (7)

Two profit functions have to be formulated for two different Cases.

Profit function for Case 1
 = Revenue from sells- Set up cost- Holding cost- Interest paid+ Interest obtain- Cost for promotional effort

$$\begin{aligned}
 \pi_1 &= \text{Average profit for Case 1} \\
 &= \frac{\text{Profit function for Case 1}}{\text{Cycle length}}
 \end{aligned}$$

Then expression for π_1 becomes

$$\pi_1 = \frac{1}{T} \left[DT (s - p) - c_s - \frac{c_h DT^2}{2} - \frac{1}{2} D(T - M)^2 p I_p + \frac{1}{2} DM^2 s I_o - k \rho^2 \right] \tag{8}$$

[Using (3), (4) and (5)]
 Similarly we obtain average profit for Case 2 and it is displayed below

$$\pi_2 = \frac{1}{T} \left[DT (s - p) - c_s - \frac{c_h DT^2}{2} + 0 + s I_o D T \left(M - \frac{T}{2} \right) - k \rho^2 \right] \tag{9}$$

[Using (3), (6) and (7)]
Solution methodology:

Since this work deals with profit maximization problem, so the corresponding necessary condition is follows as:

$$\frac{\partial \pi_i}{\partial s} = 0, \frac{\partial \pi_i}{\partial \rho} = 0 \text{ for } i = 1,2.$$

Sufficient condition is that the following Hessian matrix H is negative definite.

$$H = \begin{bmatrix} \frac{\partial^2 \pi_i}{\partial s^2} & \frac{\partial^2 \pi_i}{\partial s \partial \rho} \\ \frac{\partial^2 \pi_i}{\partial s \partial \rho} & \frac{\partial^2 \pi_i}{\partial \rho^2} \end{bmatrix} \text{ for } i = 1,2.$$

If the eigen values of the Hessian matrix H are negative then it is negative definite.

Numerical Example :

For Case1:

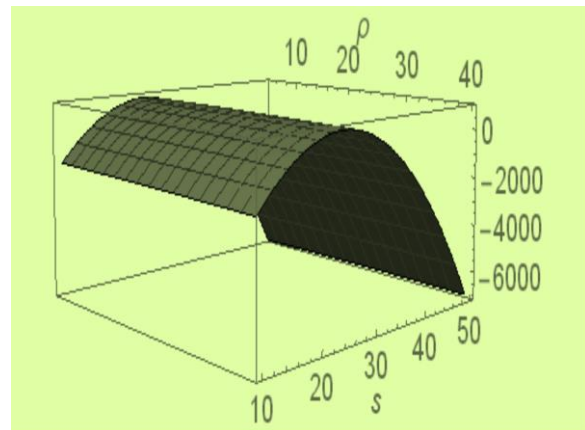
Parameters	Values
p	\$10 per unit
c_s	\$1200 per set up
ξ	0.45
c_h	\$2 per unit
M	5 days
T	8 days
I_o	0.20 per\$ per unit time
I_p	0.25 per\$ per unit time
k	0.5
a	300
b	9

Optimal results:

s^*	\$24.22 per unit
ρ^*	11.59
π_1^*	\$894.51

These results are optimal since eigen values of the matrix H at (s^*, ρ^*) are $-23.63, -0.19$.

Three dimensional representation of profit function with respect to decision variables selling price and promotional effort is given below.



For Case2:

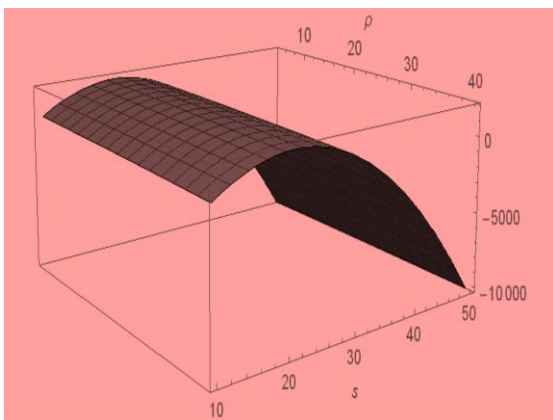
The only difference from the point of parameter values is $T = 4 \text{ days}$. Here credit period offered by the supplier to the retailer is greater than cycle length.

Optimal results:

s^*	\$21.19 per unit
ρ^*	10.06
π_2^*	\$1918.91

These results are optimal since eigen values of the matrix H at (s^*, ρ^*) are $-28.80, -0.39$.

Three dimensional representation of profit function with respect to decision variables selling price and promotional effort is given below



This section has been done by taking help of the Software MATHEMATICA.

Conclusion:

The objective of this work is to determine selling price together with level of promotional effort such that a retailer can optimize his profit in business environment. Also the effect of credit financing has been incorporated in this model. From numerical result it has been found that retailer get more profit when credit period offered by supplier exceeds his cycle length. It is quite reasonable as in this situation he has to pay no amount as interest to the supplier. Optimality of the profit function has been checked both numerically and graphically.

There are many ways of possible extension of this work. Two tier credit financing may be considered. Inclusion of perishable item in this frame work can also be done. Credit linked demand rate can be assumed for the generalization of this model.

References:

1. H. Krishnan, R. Kapuscinski, D.A. Butz , “Coordinating for decentralized supply chains with retailer promotional effort “, *Management Science*, 2004, **50(1)** , 48-63.

2. B. Pal, S. S. Sana, K. S. Chaudhuri, “ Two echelon manufacturer- retailer supply chain strategies with price, quality and promotional effort sensitive demand”, *International Transactions in Operational Research*, 2015,1071-1095.

3. B. Pal, A. K. Bhunia, K. S. Chaudhuri, “ Promotional effort and quality sensitive two echelon production inventory model with partial backlogging”, *International Journal of Mathematics in Operational Research*,2018,**13(4)**,533-547.

4. A. Mandal, B. Pal, “Optimizing profit for pricing and advertisement sensitive demand under unreliable production system”, *International Journal of System Science: Operations and Logistics*, 2021, **8(2)**, 99-118.

5. S. K. Goyal, “ Economic order quantity under conditions of permissible delay in payments”, *The Journal of the Operational Research Society*,1985, **36(4)**, 335-338.

6. S. Tiwary, L. E. Cardenas Barron, A. Khanna, C. K. Jaggi, “ Impact of trade credit and inflation on retailer’s ordering policies for non-instantaneous deteriorating items in a two-warehouse environment, *International Journal of Production Economics*, 2006, **176**,154-169.

7. B. Sarkar, B. K. Dey, M. Sarkar, S. Hur, B. Mandal, V. Dhaka, “Optimal replenishment decision for retailers with variable demand for deteriorating products under a trade-credit policy”, *RAIRO-Operations Research*, 2020, **54(6)**, 1685-1701.

8. A. Mandal, B. Pal, K. S. Chaudhuri, “ Unreliable EPQ model with variable demand under two tier credit financing”, *Journal of Industrial and Production Engineering*,2020, **37(7)**, 370-386.