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Price sensitive economic order quantity model of non-instantaneous deteriorating items with exponential partial backlogging

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Abstract:

This illustration is made on an economic order quantity model. Demand rate, which is an essential component of an inventory system, here decreases linearly with price. This framework of modeling includes shortages of goods at the beginning of the inventory cycle. The backlogging rate of shortage quantity depends on the waiting time of the customer. Exact form of backlogging rate is exponential. Inventory deteriorates at a constant rate non-instantaneously. This model features to determine optimal ordering quantity, backordered amount and selling price so that total profit is maximized. In support of this proposed model, a numerical example is exhibited.

Keywords: *Inventory, price dependent demand, shortage, partial backlogging, non-instantaneous deterioration.*

Introduction:

Problems of inventory are very much attractive to the researchers and practitioners as these types of problems are much related to the real lives. Demand rate which is an essential component of all inventory problems influenced by many things. Selling price is one of them. Generally demand rate increases with the decrease in selling price. So demand rate may be considered any monotonic decreasing function of selling price.

Deterioration in inventory of various items is a natural problem. But for many items deterioration does not start instantaneously. In those cases, deterioration starts after some time from the commencement of inventory cycle.

Occurrence of shortage is also familiar with inventory system. There are two ways of backlogging of shortages. They are complete and partial. However, partial backlogging situation is more realistic with respect to complete backlogging. As customer becomes impatient when item is out of stock. They can fulfill their requirement from other sources. In this process, a fraction of demand may be lost. This may hamper good-will of the business.

Considering all the above factors we formulate a mathematical model.

Literature Survey:

Wee [1] developed an inventory model for perishable items. In that model, the time to deterioration of items assumed to obey the Weibull distribution. San Jose et al. [2] formulated an economic order quantity model where shortage was partially backlogged as an exponential function. Abad [3] considered pricing with lot sizing problem with general time dependent deterioration rate. Das Roy et al. [4] framed an inventory model with imperfect quality items. In that formulation, stock-out situation was considered at the beginning of the cycle. During this situation a fraction of demand was adjusted through partial backlogging while the rest portion is assumed to be lost. Dye [5] discussed the effectiveness of investment in preservation technology in an inventory model where deterioration occurs non-instantaneously. Jaggi et al. [6] incorporated trade credit policy for non-instantaneous deteriorating items along with two types of storage facilities. Bardhan et al. [7] developed inventory model for non-instantaneous deteriorating items under stock-dependent demand situation.

Notations:

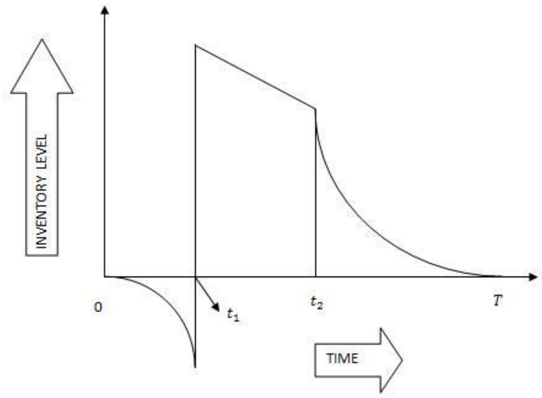
D	Demand rate
s	Selling price
c_p	purchase price
c_s	Set up cost per set up
T	Cycle length
c_h	Holding cost per unit
c_b	Backorder cost per unit
c_l	Lost sale cost per unit
c_d	Cost of deterioration per unit
θ	Deterioration rate
$I_1(t)$	Inventory level for items at time $t \in [t_1, t_2]$

$I_2(t)$	Inventory level for items at time $t \in [t_2, T]$
$S(t)$	Shortage level at time t
Q	Optimal order quantity
B	Maximum shortage amount
t_1	Shortage period
t_2	Starting time for deterioration
BAC	Total cost for back ordering
LSC	Total cost for lost sale
H_1	Total holding cost for the period $[t_1, t_2]$
H_2	Total holding cost for the period $[t_2, T]$
Q_d	Total deteriorated quantity
π_1	Total Profit
π_2	Average total profit

Basic assumptions:

- A single item is considered.
- Demand rate is a monotonic decreasing function of selling price.
- Inventory cycle starts with shortage. Shortage interval is $[0, t_1]$.
- Shortage is partially backlogged. Backlogging rate is $e^{-k\,w}$, where $k > 0$ and w is the waiting time of the customer.
- Effect of lost sale is taken into account
- Deterioration starts at time $t_2$which is known from previous experience. Here, $t_2 = t_1 + \mu$ with μ is positive real number. Rate of deterioration of inventory is constant.
- Decision variables are ordering quantity, maximum allowable shortage amount and selling price of the commodity.
- A deteriorated item has no selling value.

Mathematical Model formulation:



The model begins with shortage. Shortage continues up to the time t_1. During shortage of goods, some customer has to be lost due to the fact they may not wait. Actually, the customer arriving at time $t \in (0, t_1)$ has to wait for the time $t_1 - t$. At time t_1, replenishment is made by the quantity Q. This amount is divided into two parts. Quantity B is used as backordered quantity while the rest amount $Q - B$ is used for increasing inventory level. Deterioration starts after some time of replenishment. For the period $[t_1, t_2]$ there is no deterioration. Inventory depletes due to the demand rate only. Hazards of deterioration take place in the interval$[t_2, T]$. In this interval, inventory reduces for both of customer demand and deterioration.

Shortage level for the time interval $[0, t_1]$ is governed by the equation

$$\frac{dS}{dt} = -D\, e^{-k(t_1 - t)} \text{ subject to } S(0) = 0. \qquad (1)$$

Solution of the above differential equation using initial condition, we get

$$S(t) = \frac{D}{k}\left[e^{-k\,t_1} - e^{-k(t_1 - t)}\right] \qquad (2)$$

Using the terminal condition $S(t_1) = -B$, we obtain the expression for backorder quantity

$$B = \frac{D}{k}\,(1 - e^{-kt_1}) \qquad (3)$$

Now, the backordering cost is described as

$$BAC = c_b \int_0^{t_1} \{-S(t)\}\, dt$$

$$= c_b \left[\frac{D}{k^2} - \frac{D}{k}\left(1 - \frac{k\,B}{D}\right)\left(t_1 + \frac{1}{k}\right)\right] \qquad (4)$$

From the relation (3), the expression for t_1 is obtained as

$$t_1 = \frac{1}{k}\log\left(\frac{D}{D - k\,B}\right) \qquad (5)$$

During shortage period the demand which is unmet can be visualized as $D - D\, e^{-k(t_1 - t)}$. So the lost sale cost estimated is given below.

$$LSC = c_l \int_0^{t_1} \left[D - D\, e^{-k(t_1 - t)}\right] dt$$

$$= c_l (Dt_1 - B\ )$$

[Using the relation (3)] (6)

The differential equation governing inventory level for the time interval $[t_1, t_2]$ is given by

$\frac{dI_1}{dt} = -D$ with $I_1(t_1) = Q - B$ (7)

The solution of this differential equation is given by

$I_1(t) = Q - B - D(t - t_1)$ (8)

Holding cost for the time interval $[t_1, t_2]$ is framed as

$H_1 = c_h \int_{t_1}^{t_2} I_1(t)dt$

$= c_h \left[(Q - B)\mu - \frac{1}{2} D\mu^2\right]$, where $\mu = t_2 - t_1$ (9)

The differential equation governing inventory level for the time interval $[t_2, T]$ is given by

$\frac{d\,I_2}{dt} = -D - \theta I_2$ subject to $I_2(T) = 0$ (10)

Solving this differential equation, we have

$I_2(t) = \frac{D}{\theta}\left[e^{\theta(T-t)} - 1\right]$ (11)

As inventory level is a continuous function at $t = t_2$, then we have

$I_1(t_2) = I_2(t_2)$. From this condition and using some algebraic manipulation we get cycle time

$T = t_1 + \mu + \frac{1}{\theta} \log\left[1 + \frac{(Q-B-D\,\mu)\theta}{D}\right]$ (12)

Holding cost for the time interval $[t_2, T]$ is framed as

$H_2 = c_h \int_{t_2}^{T} I_2(t)dt$

$= c_h \left[\frac{D}{\theta^2}\{e^{\theta(T-t_2)} - 1\} - \frac{D}{\theta}(T - t_2)\right]$ (13)

$Q_d = Total\ amount\ deteriorated = \theta \int_{t_2}^{T} I_2(t)dt$

$=> Q_d = \left[\frac{D}{\theta}\{e^{\theta(T-t_2)} - 1\} - D(T - t_2)\right]$ (14)

Total sales revenue is $s(Q - Q_d)$

Total Profit $= \pi_1 = Sales\ revenue - purchase\ cost - set\ up\ cost - holding\ cost - back\ order\ cost - lost\ sale\ cost - deterioration\ cost$

$\pi_1 = s(Q - Q_d) - c_p\,Q - c_s - (H_1 + H_2) - BAC - LSC - c_d Q_d$ (15)

$\pi_2 = Average\ total\ profit = \frac{\pi_1}{T}$ (16)

Solution Procedure:

Solving three equations $\frac{\partial \pi_2}{\partial Q} = 0, \frac{\partial \pi_2}{\partial B} = 0, \frac{\partial \pi_2}{\partial s} = 0$ we get $Q = Q^*, B = B^*, s = s^*$. These solutions are optimal if eigen values of the following Hessian matrix are negative.

$$\left[\begin{matrix} \frac{\partial^2 \pi_2}{\partial Q^2} & \frac{\partial^2 \pi_2}{\partial Q\,\partial B} & \frac{\partial^2 \pi_2}{\partial Q\,\partial s} \\ \frac{\partial^2 \pi_2}{\partial Q\,\partial B} & \frac{\partial^2 \pi_2}{\partial B^2} & \frac{\partial^2 \pi_2}{\partial B\,\partial s} \\ \frac{\partial^2 \pi_2}{\partial Q\,\partial s} & \frac{\partial^2 \pi_2}{\partial B\,\partial s} & \frac{\partial^2 \pi_2}{\partial s^2} \end{matrix}\right]\Bigg|_{Q=Q^*,B=B^*,s=s^*}$$

Due to non-linearity functions, analytical solution is very complicated. We solve this problem numerically by the help of MATHEMATICA SOFTWARE.

Numerical Example:

Here demand taken is taken as $D = a - bs\ where\ a\ and\ b\ are\ positive\ real\ numbers$

Parameters	Values
c_p	\$10 per unit
c_s	\$1000 per set up
c_h	\$1.25 per unit
c_l	\$1.50 per unit
μ	0.42
θ	0.1
c_d	\$0.5 per unit
a	500
b	5

Optimal results:

Q^*	514.18 unit
B^*	76.47 unit
s^*	\$55.70 per unit

These results are optimal since eigen values of the matrix are all negative and their values are $-10.0054, -0.0272741, -0.0030601$.

Conclusion:

This work analyses an inventory system of items which deteriorates after some time. An example of these types of items may be potatoes. Here storage is back ordered partially depending upon impatient behavior of customer during stock-out situation. This model aims to maximize average profit based on three decision variables.

There are many options by which this model can be extended. This concept can be explored by considering production inventory system. Non-constant deterioration rate may be taken into consideration for another possible extension. Option for permissible delay in payment may also be introduced.

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